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MATHEMATICS.—*Note on an integrating device.* M. D. HERSEY, Bureau of Standards. (Communicated by Louis A. Fischer.)

This note offers an approximate method for evaluating the integral

$$I = \int_{x_1}^{x_2} f(y)\, dx \qquad (1)$$

in which $f(y)$ is stated analytically but in which the relation between y and x is available in the form of a curve only.

When $f(y)$ takes the form y, $\frac{y^2}{2}$, or $\frac{y^3}{3}$, the integral becomes, respectively, the area, the statical moment, or the moment of inertia, about the x-axis, of the plane figure bounded by the curve, the x-axis, and the limiting ordinates at x_1 and x_2. These three problems are familiar ones in machine design and naval architecture. They are frequently solved, with sufficient accuracy for the purpose, by plotting an auxiliary curve of squares or cubes if the case requires it, and then determining an area by cutting up the figure into strips of equal width, and applying some average ordinate rule. The present device is offered as a substitute for the latter method. It is equally accurate and more convenient. The applicability of the device is not limited to these particular functions.

The device consists of a templet, or plane figure, to be cut out of stiff paper, celluloid, or German silver. In its simplest form, the templet is bounded by two perpendicular lines and a curve. Call the two straight lines respectively the back and the base,

and call the curve the front of the templet. The templet is to be placed on the drawing board right over the (x, y) curve so that it can be slid along with its base on the x-axis. Starting with the back at x_1, make a mark where the front of the templet crosses the (x, y) curve. Then slide it along until the back comes to the mark and make a second mark where the front now crosses the curve, and so on. Let n be the number of steps necessary to travel across from x_1 to x_2 in this manner. An approximate numerical value of the integral I will then be

$$I = nC \tag{2}$$

in which C is a known constant for a given templet.

In order to obtain the simple result (2), it is necessary only that the front of the templet be cut to the curve

$$f(Y) \cdot X = C \tag{3}$$

Here X and Y are respectively the abscissa and ordinate of any point on the templet curve, relative to the back and the base as axes.

To prove (2), let the variable Δx denote the width of each step along the x-axis. When the templet is in any one of the successive positions marked on the (x, y) curve, $X = \Delta x$ and $Y = y$. Hence by (3)

$$f(y) \cdot \Delta x = C \tag{4}$$

Integrating (1) between x and $x + \Delta x$ gives for the contribution which the strip of width Δx makes to the integral I, approximately

$$\Delta I = f(y) \cdot \Delta x \tag{5}$$

Comparing (4) and (5),

$$\Delta I = C \tag{6}$$

Thus every strip contributes the same amount C; therefore the whole integral is

$$I = \Sigma \Delta I = nC \tag{7}$$

The accuracy of the result is enhanced if the device be made up of two such templets, back to back in one piece. The working formula will then be

$$I = nK \tag{8}$$

in which

$$K = 2C \tag{9}$$

Any number may be chosen for K, and the templet cut accordingly. It is desirable that K be a multiple of 10, provided this does not make the device inconveniently large or small. It is immaterial where the extremities are cut off. Further expedients for simplifying the work, such as the use of templets of graded sizes, or auxiliary base lines for the (x, y) curve, will suggest themselves upon examining each particular problem.

The equilateral hyperbola has the property that the rectangle formed under any point has a constant area. This property has been utilized in various ways for determining the areas of plane figures. One such device is known as Beauvais' hyperbolic triangle.[1] The present contribution is nothing other than a generalization of the hyperbolic triangle so that it will determine any function, and not simply areas.

The statical moment and moment of inertia have been cited as functions which can be evaluated by the new device. Another function, which the writer has met both in barometric altitude calculations and in studying the effect of pressure on viscosity, is the integral of the reciprocal of the ordinate of an empirical curve,

$$\int_{x_1}^{x_2} \frac{dx}{y}$$

The templet needed for stepping off this integral is simply an inverted triangle.

To integrate any function $F(x)$ which can be written $f[\phi(x)]$, it is necessary only to evaluate $\int f(y)dx$ along an auxiliary curve $y = \phi(x)$. The result will be $\int F(x)dx$. For example, let it be required to find $\int_{x_1}^{x_2} e^{-x^2} dx$. Here

$$\left.\begin{aligned} F(x) &= e^{-x^2} \\ \phi(x) &= x^2 \\ f(y) &= e^{-y} \end{aligned}\right\} \tag{10}$$

[1] Engineering News, 66: 340, 628. 1911.

Cut out a double faced templet to the curve $e^{-Y} \cdot X = \text{const.}$, i.e., to the curve

$$Y = \log \frac{2}{K} + \log X \tag{11}$$

in which K is chosen at pleasure. If n is the number of steps needed for traversing the curve

$$y = x^2 \tag{12}$$

from x_1 to x_2 with this templet,

$$\int_{x_1}^{x_2} e^{-x^2} dx = nK \tag{13}$$

The device therefore is not limited to problems involving empirical curves to start with. It is applicable also to cases in which the integrand is given analytically. It will be practically useful in such cases, whenever $F(x)$ is sufficiently complicated to warrant the trouble of dealing separately with the two functions $\phi(x)$ and $f(y)$.

PHYSICS.—*Note on a relation connecting the derivatives of physical quantities.*[1] M. D. HERSEY, Bureau of Standards. (Communicated by E. Buckingham.)

Statement of the problem. Given the fact that some relation of unknown form

$$Q_0 = f(Q_1, Q_2, \ldots Q_{N-1}) \tag{1}$$

subsists between N physical quantities $Q_0, Q_1, Q_2, \ldots Q_{N-1}$, no others being involved, it is required to deduce a relation of known form

$$\frac{\partial Q_0}{\partial Q_1} = F\left(\frac{\partial Q_0}{\partial Q_2}, Q_0, Q_1, Q_2, \ldots Q_{N-1}\right) \tag{2}$$

such that at any point whose generalized coordinates, Q_0, Q_1, Q_2, etc., are given, the value of any one of the N-1 partial derivatives of Q_0 can be computed from any other. Thus, it is required to calculate one of the component slopes of the generalized sur-

[1] This work was done at the Jefferson Physical Laboratory, Harvard University.

face (1) from a knowledge of another, although the equation of the surface is not available. The interest of the problem to the physicist lies in the fact that he may wish to learn the value of a derivative not readily accessible to experiment, in a case where some other derivative of the same quantity can easily be observed. It will be shown that a definite solution can always be obtained, provided certain dimensionless products of the N quantities are held constant.

Other classes of relations among derivatives. The proposition, that relations may be found connecting the derivatives of quantities in the absence of a primitive equation, is not new. There are two other classes of such relations. One consists of mathematical identities, applicable to any set of related quantities, whether physical or not. To this class belongs the identity

$$\frac{\partial}{\partial Q_1}\frac{\partial Q_0}{\partial Q_2} = \frac{\partial}{\partial Q_2}\frac{\partial Q_0}{\partial Q_1} \tag{3}$$

as well as the triple product relation

$$\frac{\partial Q_0}{\partial Q_1} \cdot \frac{\partial Q_1}{\partial Q_2} \cdot \frac{\partial Q_2}{\partial Q_0} = -1 \tag{4}$$

The other class comprises relations requiring the explicit use of physical laws, such as the two laws of thermodynamics, or Hamilton's principle. To this class belong Maxwell's four thermodynamic relations, and the reciprocal relations of generalized dynamics.[2] The relations to be presented here are of a nature intermediate between the other two classes, in that they require a knowledge only of the dimensions of the quantities.

Derivation of the new relation. The present result depends upon and is a corollary to Buckingham's Π-theorem,[3] according to which any complete physical equation is reducible to the form

$$\text{funct.}(\Pi_1, \Pi_2, \ . \ . \ . \ \Pi_i) = 0 \tag{5}$$

[2] J. J. THOMSON, Applications of dynamics to physics and chemistry, Chap. V.

[3] This JOURNAL, **4**: 347–353. 1914; Phys. Rev., **4**: 345–376. 1914; Trans. Am. Soc. Mech. Engs., **37**: 263–296. 1915. Any one who can sufficiently visualize the meaning of the Π-theorem will be able to treat each concrete problem by itself, dispensing with the formulas of the present paper save as a check.

in which the Π's are all the independent dimensionless products which can be built up by combining in any way the N physical quantities involved. Further, the total number of such products, or dimensionless arguments, will always be the same, no matter how the quantities are grouped. This number will be

$$i = N - k \tag{6}$$

if k is the number of fundamental units needed for measuring the N quantities.[4]

Let Π_0 and Π designate any two of the i products in (5) which contain between them the three quantities Q_0, Q_1, and Q_2 in which we are interested. Let Q_0 appear to the first power in Π_0 and not at all in any other product. This can always be done, for Buckingham has shown that a certain standard arrangement is possible in which each product contains to the first power some one quantity of type P which occurs nowhere else.[5] We shall then have

$$\Pi_0 = Q_1^{\alpha_0} Q_2^{\beta_0} \cdots Q_k^{\kappa_0} \cdot Q_0 \tag{7}$$

and

$$\Pi = Q_1^{\alpha} Q_2^{\beta} \cdots Q_k^{\kappa} \cdot Q_{k+1} \tag{8}$$

The exponents are abstract numbers fixed by the dimensions of the N quantities; in any particular problem some of them may be zero. If we now agree to keep the remaining $i-2$ products constant, (5) becomes

$$\Pi_0 = \phi(\Pi) \tag{9}$$

in which the form of ϕ is unknown. The restriction to constant products can always be fulfilled in theory, but it may lead to difficulties in practice; it will be discussed in a later section. Differentiating (9) and then (8) gives in succession

$$\frac{\partial \Pi_0}{\partial Q_1} = \frac{d\phi}{d\Pi} \cdot \frac{\partial \Pi}{\partial Q_1} = \frac{d\phi}{d\Pi} \frac{\alpha \Pi}{Q_1} \tag{10}$$

From (7)

$$\frac{\partial \Pi_0}{\partial Q_1} = \frac{\partial Q_0}{\partial Q_1} \frac{\Pi_0}{Q_0} + \frac{\alpha_0 \Pi_0}{Q_1} \tag{11}$$

[4] The question of the number of fundamental units needed has been discussed by Riabouchinsky, Rayleigh, and Buckingham; see Nature, **95**: 396–397. 1915.

[5] Trans. Am. Soc. Mech. Engs., **37**: 291–292; note eq. (11) and its discussion.

Comparing (10) and (11)

$$\Pi \frac{d\phi}{d\Pi} = \frac{Q_1 \Pi_0}{\alpha}\left(\frac{1}{Q_0}\frac{\partial Q_0}{\partial Q_1} + \frac{\alpha_0}{Q_1}\right). \tag{12}$$

Similarly

$$\Pi \frac{d\phi}{d\Pi} = \frac{Q_2 \Pi_0}{\beta}\left(\frac{1}{Q_0}\frac{\partial Q_0}{\partial Q_2} + \frac{\beta_0}{Q_2}\right) \tag{13}$$

Comparing (12) and (13)

$$\frac{\partial Q_0}{\partial Q_1} = \left(\frac{\alpha}{\beta}\beta_0 - \alpha_0\right)\frac{Q_0}{Q_1} + \frac{\alpha}{\beta}\frac{Q_2}{Q_1}\frac{\partial Q_0}{\partial Q_2} \tag{14}$$

Hence the desired relation (2) has the linear form

$$\frac{\partial Q_0}{\partial Q_1} = a + b\frac{\partial Q_0}{\partial Q_2} \tag{15}$$

in which the coefficients

$$a = \left(\frac{\alpha}{\beta}\beta_0 - \alpha_0\right)\frac{Q_0}{Q_1} \tag{16}$$

and

$$b = \frac{\alpha}{\beta}\frac{Q_2}{Q_1} \tag{17}$$

involve none of the N quantities save Q_0, Q_1, and Q_2.

Evidently (14) can be written also

$$\frac{\partial \log Q_0}{\partial \log Q_1} = \left(\frac{\alpha}{\beta}\beta_0 - \alpha_0\right) + \frac{\alpha}{\beta}\frac{\partial \log Q_0}{\partial \log Q_2} \tag{18}$$

in which the coefficients are independent of the coordinates. Thus the relation connecting the logarithmic derivatives is the same all over the generalized surface.

Extension to higher derivatives. Differentiating (14) with respect to Q_1 and using the identity (3) gives

$$\frac{\partial^2 Q_0}{\partial Q_1^2} = A + B\frac{\partial Q_0}{\partial Q_2} + C\frac{\partial^2 Q_0}{\partial Q_2^2} \tag{19}$$

in which the coefficients are

$$A = \frac{Q_0}{Q_1^2}\left(\frac{\alpha}{\beta}\beta_0 - \alpha_0\right)\left(\frac{\alpha}{\beta}\beta_0 - \alpha_0 - 1\right) \tag{20}$$

$$B = \frac{Q_2}{Q_1^2}\frac{\alpha}{\beta}\left(\frac{\alpha}{\beta} - 1\right)(1 + 2\alpha_0) \tag{21}$$

and

$$C = \left(\frac{Q_2\,\alpha}{Q_1\,\beta}\right)^2 \tag{22}$$

Thus the curvature with respect to Q_1 can be calculated from the slope and the curvature with respect to Q_2.

Integral form of the relation.[6] Integrating (14) at the point ($Q_0 = q_0$, $Q_1 = q_1$, $Q_2 = q_2$) over an interval so short that $\frac{\partial Q_0}{\partial Q_2}$ may be treated as constant, and denoting its value by the symbol $\frac{\partial q_0}{\partial q_2}$, gives for the primitive equation of an element of the surface

$$\frac{Q_0}{q_0} = \left(\frac{Q_1}{q_1}\right)^h \tag{23}$$

in which

$$h = \frac{\alpha}{\beta}\left(\frac{q_2}{q_0}\frac{\partial q_0}{\partial q_2} + \beta_0\right) - \alpha_0 \tag{24}$$

The use of (23) would permit a direct comparison of any new results obtained by the present method with empirical results previously published in one-term, constant-exponent formulas.

Discussion of the constant-product restriction. Let Π_c denote any one of the $i-2$ arguments which we have agreed to hold constant, and let Q stand for either Q_1 or Q_2. Then, unless Π_c can be so chosen that it does not contain Q, it must be so chosen that it will contain some additional quantity Q_c not occurring

[6] If instead of an isolated value of $\frac{\partial Q_0}{\partial Q_2}$ we were furnished with the entire curve $Q_0 = f_2(Q_2)$, the direct use of the Π-theorem would be preferable, and would give the whole curve $Q_0 = f_1(Q_1)$. If successively furnished with additional curves, $Q_0 = f_3(Q_3)$ and so on, we could gradually build up generalized cross sections of the surface (1) until, when N-k independent curves had been given, we should have the whole of it. The problem of developing empirical equations synthetically has not been treated in the available papers. That problem is a general one, of which the problem of the present paper is a special case; this situation is illustrated by the fact that our final result (23) applies only to an infinitesimal piece of the curve $Q_0 = f_1(Q_1)$.

in any other product. The rule for keeping Π_c constant will then be: Vary Q_c simultaneously in such a manner as to compensate the changes due to Q.

If Q enters Π_c to the n^{th} power and Q_c enters it to the first, the derivatives in (15) and elsewhere are subject to one or more conditions of the type $Q_c \propto Q^{-n}$. For such a derivative let us adopt from now on the notation $\left(\frac{\partial Q_0}{\partial Q}\right)_{Q_c \propto Q^{-n}}$. There are two experimentally independent methods for getting its numerical value: First, by directly observing the change in Q_0 with Q while simultaneously changing Q_c in the prescribed manner; second, by calculating it from separate observations on the change in Q_0 with Q at constant Q_c, and the change in Q_0 with Q_c at constant Q. Expanding the conditioned derivative into the form $\left(\frac{\partial Q_0}{\partial Q}\right)_{Q_c} + \left(\frac{\partial Q_0}{\partial Q_c}\right)_Q \frac{dQ_c}{dQ}$ and taking account of the fixed relation between Q_c and Q leads to the working formula

$$\left(\frac{\partial Q_0}{\partial Q}\right)_{Q_c \propto Q^{-n}} = \left(\frac{\partial Q_0}{\partial Q}\right)_{Q_c} - n\frac{Q_c}{Q}\left(\frac{\partial Q_0}{\partial Q_c}\right)_Q \tag{25}$$

for the second method. In the most general case where there are $i-2$ arguments to be kept constant, the second term on the right of (25) will be replaced by $-\frac{1}{Q}$ times the summation of $i-2$ terms of the type $nQ_c\left(\frac{\partial Q_0}{\partial Q_c}\right)_Q$.

While the procedure outlined in this section is always possible and sufficient, it is not always necessary or even desirable. For example: if the number of quantities, N, does not exceed the number of fundamental units, k, by more than 2, there will be no other arguments than Π_0 and Π; again, if the remaining $i-2$ arguments do not involve Q (i.e., Q_1 or Q_2), their constancy will not be disturbed at all by the fact that Q_1 and Q_2 do vary. Further expedients for simplifying the work will suggest themselves upon examining each particular case by itself.

Some illustrative examples. For reference in solving problems it is convenient to rewrite (5) in the form

$$Q_1^{\alpha_0} Q_2^{\beta_0} \cdots Q_k^{\kappa_0} \cdot Q_0 = \text{funct.}\,(Q_1^{\alpha} Q_2^{\beta} \cdots Q_k^{\kappa} \cdot Q_{k+1}, \text{ and other } \Pi\text{'s}) \quad (26)$$

The values of α, β, etc, can now be read off directly by identifying them with the corresponding numerical exponents in the equation, of type (26), afforded by the particular example in hand.

I. In the case of a journal bearing, under certain restrictions, we may expect a relation of type (1) to connect the coefficient of friction f, with the viscosity of the lubricant μ, the revolutions per unit time n, the bearing pressure p, the journal diameter D, and the volume of oil V forced through the bearing in unit time. Let it be required to calculate the effect of altering the size of the machine from a test in which nothing is varied but the rate of pumping oil through the bearing. By the Π-theorem,

$$f = \text{funct.}\left(\frac{D^3 n}{V}, \frac{\mu n}{p}, \text{shape}\right) \quad (27)$$

Let f, D and V serve respectively as Q_0, Q_1, and Q_2. Comparing (27) with (26), $\alpha_0 = 0$, $\beta_0 = 0$, $\alpha = 3$, $\beta = -1$; hence, by (16) and (17), $a = 0$ and $b = -3\dfrac{V}{D}$, or

$$\frac{\partial f}{\partial D} = -3\frac{V}{D}\frac{\partial f}{\partial V} \quad (28)$$

Also, by (20) and (22), $A = 0$, $B = 12\dfrac{V}{D^2}$, and $C = 9\left(\dfrac{V}{D}\right)^2$; therefore

$$\frac{\partial^2 f}{\partial D^2} = 12\frac{V}{D^2}\frac{\partial f}{\partial V} + 9\left(\frac{V}{D}\right)^2 \frac{\partial^2 f}{\partial V^2} \quad (29)$$

Equations (28) and (29) enable us to predict the bearing losses of any slightly larger or smaller machine in the same geometrically similar series. This requirement of geometrical similarity is an instance of the constant-product restriction. The products in this case are the length ratios fixing the shape.

II. Let it be required to find the effect of gravity on a rolling ball viscosimeter in terms of the effect produced by changing

the size of the instrument. Let D, l, and θ denote, respectively, the diameter and length of the tube and its angle of inclination to the horizontal, d and ρ_0 the diameter and density of the ball, ρ and μ the density and viscosity of the liquid, and t the roll-time[7] in a locality[8] of gravity g. Assuming that a complete relation does subsist among these quantities, the Π-theorem shows that any equation describing that relation, whether obtained theoretically or experimentally, must be reducible to the form

$$\frac{\mu}{\rho D^2} t = \text{funct.}\left(\frac{\rho_0}{\rho}, \frac{g\rho^2 D^3}{\mu^2}, \text{shape}\right) \tag{30}$$

the shape, in turn, being fixed by the arguments $\frac{d}{D}$, $\frac{l}{D}$, and θ. Taking t, g, and D respectively for Q_0, Q_1, and Q_2 gives $\alpha_0 = 0$, $\beta_0 = -2$, $\alpha = 1$, and $\beta = 3$; so that by (18)

$$\frac{g}{t}\frac{\partial t}{\partial g} = -\frac{2}{3} + \frac{1}{3}\frac{D}{t}\frac{\partial t}{\partial D} \tag{31}$$

An interesting check on (31) is afforded by differentiating the empirical equation for such an instrument.[9] The equation has been presented in the form $y = a + bx$, in which x denotes $\tau\sqrt{Dg\left(\frac{\rho_0}{\rho} - 1\right)}$ and y denotes $\nu\Big/\sqrt{D^3 g\left(\frac{\rho_0}{\rho} - 1\right)}$, τ being the roll-time per unit length $\frac{t}{l}$, ν the kinematic viscosity $\frac{\mu}{\rho}$, and a and b particular numerical values fixed by a particular choice of $\frac{d}{D}$ and θ. Recast in the form (30) it becomes

[7] That is, the time required for the ball to roll down. This instrument, proposed by Flowers (Proc. Am. Soc. Test. Mat., **14**: 565. 1914), is further discussed by the writer in this JOURNAL, **6**: 527. 1916.

[8] Having set up such a viscosimeter in Cambridge, the question arose whether there would be any sensible change upon taking it to Washington, where gravity is 0.3 per cent less. The conclusion is that the roll-time in a very viscous liquid will be 0.3 per cent greater in Washington; and that the effect of gravity diminishes when the fluidity of the liquid increases, falling to 0.2 per cent for water.

[9] This JOURNAL, **6**: 528, eq. (6). 1916.

$$\frac{\mu}{\rho D^2} t = \frac{1}{b} \cdot \frac{1}{\frac{\rho_0}{\rho} - 1} \left(1 - a \sqrt{\frac{\rho_0}{\rho} - 1} \sqrt{\frac{g\rho^2 D^3}{\mu^2}}\right) \frac{\mu^2}{g\rho^2 D^3} \cdot \frac{l}{D} \tag{32}$$

or

$$t = \frac{A}{gD} (1 + B\sqrt{gD^3}) \tag{33}$$

in which A and B (both intrinsically positive) do not involve g at all, nor D except in a shape factor. The values of $\frac{g}{t} \frac{\partial t}{\partial g}$ and $\frac{D}{t} \frac{\partial t}{\partial D}$ found by differentiating (33) do satisfy (31).

III. Without knowing the empirical equation let it be required to predict the change in roll time due to any small change in liquid density, such as would occur upon using the tube under pressure, by reference to an observation on the effect of changing the ball density. Since an expression for $\frac{\partial t}{\partial \rho}$ in terms of $\frac{\partial t}{\partial \rho_0}$ is sought, t, ρ, and ρ_0 are selected for Q_0, Q_1, and Q_2 respectively. If (30) were to be used as it stands there would be a restriction on the derivative $\frac{\partial t}{\partial \rho}$, which is hardly to be desired. An equivalent result in a more convenient form can evidently be obtained by confining ρ to a smaller number of arguments. This is done by replacing (30) by one of the alternative forms provided by the Π-theorem, such as

$$\sqrt{\frac{g}{D}}\, t = \text{funct.} \left(\frac{\rho_0}{\rho}, \frac{g\rho_0^2 D^3}{\mu^2}, \text{shape}\right) \tag{34}$$

Comparing this with (26), $\alpha_0 = 0, \beta_0 = 0, \alpha = 1, \beta = -1$; hence by (14)

$$\frac{\partial t}{\partial \rho} = -\frac{\rho_0}{\rho} \left(\frac{\partial t}{\partial \rho_0}\right)_{\mu \propto \rho_0} \tag{35}$$

or by (25)

$$\frac{\partial t}{\partial \rho} = -\frac{1}{\rho} \left(\rho_0 \frac{\partial t}{\partial \rho_0} + \mu \frac{\partial t}{\partial \mu}\right) \tag{36}$$

In the last transformation μ took the part of Q_c and ρ_0 of Q, while n had the value -1.

The following observations afford an experimental illustration of (36). They were made with a tube 59 cm. long and 1 cm. in diameter, containing a $\frac{1}{4}$ inch (0.635 cm.) ball, ordinarily of steel ($\rho_0 = 7.7$ g./cm.^{3}). The tube was filled with lard oil ($\mu = 0.74$ c. g. s. units, $\rho = 0.92$ g./cm.^{3}). The slope $\frac{\partial t}{\partial \mu}$ was found to be 31 c. g. s. units. Substituting now a brass ball ($\rho_0 = 8.6$ g./cm.^{3}) for the steel one, the roll-time dropped from 27.9 to 24.7 seconds, making $\frac{\partial t}{\partial \rho_0}$ equal to -3.6 c. g. s. units. From these data, in conjunction with (36), the value $\frac{\partial t}{\partial \rho} = 5.2$ c. g. s. units would be predicted. From (32), the actual value is found to be 5.7 c. g. s. units. Since $\frac{\partial t}{\partial \rho}$ is itself a correction term, the agreement is sufficient.

BOTANY.—*Ammocodon, a new genus of Allioniaceae, from the southwestern United States.*[1] PAUL C. STANDLEY, National Museum.

The genus *Selinocarpus* was proposed by Gray, in 1853,[2] in a paper dealing with the plants of the family Allioniaceae[3] collected by Charles Wright during his explorations of western Texas, southern New Mexico and Arizona, and northeastern

[1] Published by permission of the Secretary of the Smithsonian Institution.

[2] Amer. Journ. Sci. II. **15**: 262.

[3] Dr. Gray used the family name Nyctaginaceae, a term more widely employed by botanists than the earlier Allioniaceae. The designation of this family is not based, as some suppose, upon the genus *Nyctaginia*, but upon *Nyctago*, an early name for the four-o'clocks, to which Linnaeus assigned the generic term *Mirabilis*, which is universally used today. Consequently the term Nyctaginaceae is objectionable, as applied to a family, since it is based upon a generic name nowhere accepted as valid.

An example of mistaken ideas concerning the source of the word Nyctaginaceae and certain related forms is found in *Catalogue of the Flowering Plants and Ferns of Connecticut* (Connecticut Geol. and Nat. Hist. Surv. Bull. 14, p. 172. 1910). In explanation of the specific name of *Oxybaphus nyctagineus* (Michx.) Sweet

Mexico, which extended from 1849 to 1852. Two species were described, *S. diffusus* and *S. chenopodioides*. There is no indication that the genus was based primarily upon either species; consequently the first, *S. diffusus*, may be taken as the type.

Selinocarpus is related to the large genus *Boerhaavia*, being distinguished chiefly by the broad, thin wings of the fruit. In the latter, it is true, the fruit is sometimes winged, but the wings are narrow, thick, and usually veined. No one, apparently, has questioned the claims of *Selinocarpus* to generic rank, for the plants are decidedly different in their general aspect from the group of species comprised in *Boerhaavia*, as restricted by the present writer.[4]

Since 1853 five species of *Selinocarpus* have been published, the genus now being known to range from Nevada and southern Utah to western Texas and southward to Coahuila, Mexico. Upon close inspection of the seven species it is evident that one of the two original ones, *S. chenopodioides*, differs in certain floral characters from the genotype and the five subsequent additions to the genus. Its perianth is campanulate and conspicuously constricted above the ovary, while in *S. diffusus* and the other species the perianth is tubular-funnelform and not at all constricted. In the case of the latter group of species the perianth varies markedly, however, in shape and size, being only 1 cm. long and with a short tube in *S. angustifolius* Torr., and 2.5 to 4.5 cm. long, with a slender, elongate tube, in the other species. In *S. chenopodioides* the perianth is 4 to 5 mm. long. In the last, moreover, the stamens are 2 or rarely 3, their filaments free from the perianth, while in *S. diffusus* and its allies the stamens are 5 or 6, their filaments adherent to the perianth tube. These striking differences in the perianth and androecium are accompanied by habital differences, also: In *S. chenopodioides* the flowers are aggregated in many-flowered, umbelliform cymes,

(*Allionia nyctaginea* Michx.) the statement is made that it signifies "like *Nyctaginia*, a genus of this family." As a matter of fact, Michaux's species was published many years before the generic name *Nyctaginia*. His specific name doubtless alludes to the resemblance of the leaves of the *Allionia* to those of the common four-o'clock, *Mirabilis jalapa*, the *Nyctago* of pre-Linnaean botanists.

[4] Contr. U. S. Nat. Herb., **12**: 372–387. 1909.

each flower subtended by one or rarely 2 bracts, while in the other species the few flowers are solitary or geminate in the leaf axils, each subtended by 2 or 3 bracts.

In 1913 Dr. Anton Heimerl, an eminent Austrian botanist well known for his studies of this family of plants, pointed out[5] these differences and used them as a basis for the division of *Selinocarpus* into two sections, *Breviflori* and *Tubiflori*. To the writer, however, it seems that the section *Breviflori* deserves generic rank, and the name *Ammocodon* is accordingly proposed for it. The primary characters upon which the genus is based are those of the flower and androecium, and they are certainly of greater significance than the quantitative fruit characters which are used to separate *Selinocarpus* and *Boerhaavia*.

Ammocodon Standley, gen. nov.

Erect or decumbent perennial herbs with thick roots and dichotomous pubescent stems. Leaves opposite, petiolate, those of a pair often unequal, the blades succulent. Flowers umbellulate, the umbellules in open cymes, each flower subtended by a minute subulate bract, or a second smaller bract rarely also present; perianth campanulate, purplish red, constricted above the ovary, shallowly 5-lobed, the lobes plicate. Stamens 2 or rarely 3; filaments filiform, short-connate at the base, free from the perianth. Ovary narrowly oblong; style filiform, exserted; stigma peltate, smooth. Fruit a compressed anthocarp, broadly 5-winged vertically, the wings hyaline. Testa of the seed adherent to the pericarp; embryo conduplicate, the cotyledons enclosing the farinaceous endosperm; radicle elongate, descending.

Type species, *Selinocarpus chenopodioides* Gray.

Ammocodon chenopodioides (Gray) Standley.

Selinocarpus chenopodioides Gray, Amer. Journ. Sci. II. **15**: 262. 1853.

The type was collected by Charles Wright in valleys from Providence Creek to the Rio Grande, western Texas. The species ranges from western Texas through southern New Mexico to southeastern Arizona, and southward into Chihuahua. It is very abundant in the region about El Paso, growing chiefly in the loose sandy soil of the mesas, usually along with creosote bush (*Covillea glutinosa*). The flowers are not very showy, but bright-colored and borne in great profusion. Like those of most, if not all, of the herbaceous members of the family, they open late in the evening and close about noon or earlier the following day.

[5] Oesterr. Bot. Zeitschr., **63**: 354–355.

ABSTRACTS

Authors of scientific papers are requested to see that abstracts, preferably prepared and signed by themselves, are forwarded promptly to the editors. Each of the scientific bureaus in Washington has a representative authorized to forward such material to this journal and abstracts of official publications should be transmitted through the representative of the bureau in which they originate. The abstracts should conform in length and general style to those appearing in this issue.

TECHNOLOGY.—*An investigation of cartridge enclosed fuses. Report of the Bureau of Standards in the case of Economy Fuse and Manufacturing Co. vs. Underwriters' Laboratories (Inc.) concerning the fire and accident hazard of the Economy Refillable Fuse as compared with approved fuses.* E. B. ROSA, H. B. BROOKS, B. MCCULLOM, W. J. CANADA, and F. W. GLADDING. Bureau of Standards Technologic Paper No. 74. Pp. 199. 1916.

This report represents the results of the investigation carried out by the Bureau of Standards acting as referee on the joint request of the Economy Fuse and Mfg. Co. and Underwriters' Laboratories, Inc., on the question of the relative fire and accident hazard of Economy Refillable fuses and fuses at present listed as standard by Underwriters' Laboratories, Inc. The evidence on which the finding of the Bureau was based includes a large number of tests of fuses under widely different conditions, as well as inspections of numerous fuse installations in practice, personal interviews with many fuse users, evidence and arguments submitted by the Economy Fuse and Mfg. Co. and Underwriters' Laboratories both at a public hearing and by correspondence, and evidence and arguments submitted by a number of manufacturers of fuses at present listed as standard by Underwriters' Laboratories. The investigation disclosed that the experience with the present type of Economy fuse is not yet sufficient to determine whether the total hazard is greater or less than it is with approved fuses as they are actually used in practice. The report contains numerous tables and 110 oscillographic records showing the performance of both Economy and approved fuses under various short circuit conditions.

B. McC.

PROCEEDINGS OF THE ACADEMY AND AFFILIATED SOCIETIES

THE PHILOSOPHICAL SOCIETY OF WASHINGTON

The 775th meeting was held on May 27, 1916, at the Cosmos Club. President BRIGGS in the chair; 47 persons present. The minutes of the 774th meeting were read in abstract and approved.

The evening was devoted to a symposium on the atom. Mr. H. L. CURTIS presented a paper on *The atom as a miniature solar system.* The author briefly sketched the history of atomic theory from Dalton's work in 1803 to the recent work of J. J. Thomson, Rutherford, Nicholson, Bohr, Van der Brock, Zeeman, and others. Two types of nucleus atom are possible, viz, the planetary type and the Saturnian type. In the planetary type each electron has an orbit different from that of any other electron, as is the case of the planets rotating about the sun, but different from this in that they repel each other while the planets attract. In the Saturnian type, which is most generally accepted, the electrons rotate in rings around the nucleus. Bohr's assumptions of the laws holding at atomic dimensions and the results from his hypothesis were given in detail. The present trend of thought is towards accepting the Saturnian type of the nuclear atom. It is generally conceded that the forces which bind the parts of the atom together are different from those with which we are accustomed to deal. The radiation giving the lines of the visible spectrum is concerned with the outer rings of the atomic system, while X-rays are produced by vibrations of the inner rings of electrons. Radioactive phenomena and chemical affinity appear to be concerned with the nucleus.

Discussion. Mr. AGNEW referred to experiments in magnetization which indicate the validity of the Saturnian or planetary-type theory. Bohr's theory predicted that certain lines of the spectrum of helium were due to hydrogen. Mr. BAUER referred to the looseness of terms found in writings on the atomic theories; for example, 8 out of 10 will use "rotation" instead of "revolution." In many theories of astronomy it is not necessary to take account of rotation, but no astronomer would attempt to explain all facts and phenomena only by revolution; it, therefore, appears that the time may come when it may be necessary to consider both rotation and revolution in connection with atomic theories. Mr. SOSMAN referred to the recent work on the valence of atoms in chemical compounds. Mr. WRIGHT referred to studies in crystal structure in which particular directions within an atom find expression in atomic arrangement of crystals.

Mr. SWANN referred to conflicting theories between the chemist and the physicist with reference to the structure of the atom. He thought that perhaps some part of the apparent excellent agreement of constants might be due to a juggling of the 2π-factor in the computations. Mr. DELLINGER thought that Mr. Swann's last remark explained some agreements found in recent contributions.

Mr. W. J. HUMPHREYS presented a paper on *The magnetic field of an atom*. Recent investigations by Weiss, Ritz, Humphreys, Oxley, and Merritt of atomic phenomena and structure were reviewed. These investigations all give varied evidence in favor of the assumption that atoms have powerful magnetic fields which are of the order 10^8 gauss, and which are due, presumably, to orbital revolutions of electrons. It might seem that atoms with such strong magnetic fields would collapse; the author's calculations, however, show that the electric forces between the portions of atomic models of the Saturnian type would be more than sufficient to prevent collapse through the interaction of their powerful magnetic fields.

Mr. SWANN thought the indicated order of magnitude of the atomic fields is large, judging, for example, from computations, assuming a field of 10^8 gauss, of the moment of the equivalent magnet and of the deflection that would be produced in shooting α-particles through a thin piece of magnetic iron.

The 776th meeting was held on October 14, 1916, at the Cosmos Club. Vice-President BUCKINGHAM in the chair; 65 persons present. The minutes of the 775th meeting were read in abstract and approved.

Mr. A. L. DAY presented a communication, illustrated by lantern slides, on *Do volcanoes offer evidence in regard to the interior of the earth?* Recent studies are helping to emphasize more and more sharply the conclusion that volcanoes are local phenomena of very limited significance in affording information concerning the interior of the earth. This view is supported by the differences in chemical composition between lava outflows in different parts of the world, by the differences in altitude of the points of outflow even in neighboring volcanoes, by the apparently complete independence of one another of volcano vents which are immediately contiguous, as at Stromboli, Hawaii, and other places, and by the fact that most of the volcanic phenomena appear to derive their energy from gas reactions in which only the gases appear to be of deep-seated origin. The formation of vertical conduits, under this view, is then simply the result of gas reactions ("gas fluxing") which generate sufficient heat to melt the adjacent rock masses and to form more or less vertical outlets for gaseous or liquid products. Neighboring conduits of this kind often show complete independence of action in time, in pressure (as shown by the lava level), and in character of explosive or other activity at the mouth, all of which point to the independence of the local supply chambers to which the vents serve as outlets. Of course, a great rift like the one on the south flank of Mauna Loa, from which two lava streams recently

flowed to a distance of 8 miles, indicates a basin of much larger magnitude than those at Stromboli, but still vanishingly small when compared with the magnitude of the earth or even with the magnitude of the volcanic island (Hawaii) of which it forms a part. In fact, the complete absence in the geological record, of any really great outpouring of lava, and the absence of evidence of extreme temperatures in those outlets which have been accessible to study, point to the conclusion that all are local and probably not even deep-seated phenomena.

Discussion. Mr. WASHINGTON called attention to the characteristics of the volcanic rocks from the continent of Africa which are quite different from those, for example, from Vesuvius and Etna. Mr. CLARKE referred to the pioneer work of Herbert Spencer regarding the condition of the interior of the earth. Mr. FARQUHAR made inquiry regarding the temperature gradients determined from borings and the relation of such gradients to Chamberlin's hypothesis. Mr. DAY stated that, judging from the temperature gradient determined from borings, the interior temperature may reach 20,000° which Chamberlin admits in his hypothesis. He stated that bore-hole temperature records must always be carefully used since conditions are generally not typical; determinations of temperature gradients from different sources vary by 100 per cent.

Mr. L. A. BAUER presented a communication, illustrated by lantern slides, entitled *Concerning the origin of the earth's magnetic field.* The various recent theories regarding the origin of the Earth's magnetic field were reviewed with particular reference to their bearings on the general topic of the evening: the constitution of the earth's interior. The hypothesis of chief interest in this connection, namely, that of an iron core being the cause of terrestrial magnetism, has inherent in it many difficulties, which, however, may not be insuperable. Should experiments decisively show that increased pressure elevates the critical temperature of magnetization, then the depth of 10 to 12 miles, now supposed to limit the presence of materials in the magnetic stage, would be increased. However, the few experiments available indicate that increased pressure lowers the critical temperature of magnetization. The various hypotheses as to the earth's magnetic field being caused by electric currents within the earth's crust, or as to its connection in some manner with the speed and direction of rotation of the earth, were briefly discussed. The exceedingly small effect to be observed renders conclusive laboratory experiments, if not a hopeless task, certainly a very difficult one with present appliances. The author reiterated a belief, already expressed on a former occasion, that our chief hope at present of determining the origin of the earth's magnetic field appears to lie in the direction of determining what causes the field to vary in the remarkable manner it does. The definite limitations imposed by the variations in the earth's magnetic field, both of the periodic and aperiodic kind, and the departures of the field from the simple uniform type, are too frequently overlooked by theorists. Most theories, for example, are found inadequate when the attempt is

made to explain, besides the origin of the field, the secular variation as it is actually observed.

In conclusion it was pointed out that the solution of some of the questions entering into the problem of the origin of the earth's magnetic field must be deferred until the completion of the magnetic survey of the earth now in progress under the auspices of the Carnegie Institution of Washington.

Discussion. Mr. BURGESS discussed the question of the effect of pressure on magnetic properties of iron. Our knowledge of variations in the magnetic conditions of ferrous materials is at present insufficient to give much help. Mr. SWANN referred to the difficulty associated with the assumption of electric currents as the origin of the earth's magnetic field primarily in the explanation of the e. m. f. to which these currents owe their origin; this difficulty is not, however, as great as may appear, for it is not improbable that the state of equilibrium in the rotating earth might involve relative motion between the electrons and the ordinary matter as a condition for the absence of degradation into heat of such motion as exists. It is a known fact that the intensity of magnetization produced in a sphere of infinite permeability when placed in a magnetic field is only four times the intensity which would be produced were the permeability only two; the reason for this is to be found in the demagnetizing force which a magnetic sphere produces in its own substance. It appears that the principle inherent in the phenomenon is not limited to the case where the magnetizing influence is an ordinary magnetic field but is of wide application, so that we may say in general that it is impossible to produce, no matter what the material of the sphere may be, an appreciable magnetization in a sphere by feeble influence. J. A. FLEMING, *Secretary.*

BOTANICAL SOCIETY OF WASHINGTON

The 112th regular meeting of the Botanical Society of Washington was held in the Assembly Hall of the Cosmos Club, Tuesday, April 4, 1916. Fifty-two members and five guests were present. HARRY R. FULTON, GEORGE L. KEENAN, LESTER A. ROUND, J. F. CLEVENGER, C. E. TEMPLE, A. E. ALDOUS, VICTOR BIRCKNER, and FORREST S. HOLMES were elected to membership. The following papers were presented.

Botanical explorations in South America: J. N. ROSE.

Plants domesticated in Peru: O. F. COOK. Mr. Cook gave a brief account of the agriculture of the Incas, with their wonderful terraces and system of irrigation. Among the plants domesticated by them were maize, beans, lima beans, peanuts, quinoa (*Chenopodium quinoa*), red peppers (*Capsicum*), mandioca, tomatoes, passion fruits, sweet potatoes, tuberous *Tropaeolum* and *Oxalis*, arracacha (a celery-like plant), squashes and pumpkins, gourds; and among the fruits were chirimoyas, lucumas, and pepinos. The narcotic coca, from which cocaine is now prepared, was also grown. Mr. Cook's paper is embodied in an article since published in the National Geographic Magazine, **29**: 474–534. June, 1916.

W. E. SAFFORD, *Corresponding Secretary.*